IMPLIED RISK ADJUSTED DISCOUNT RATES AND CERTAINTY EQUIVALENCE IN CAPITAL BUDGETING

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ABSTRACT

Incorporating risk into the capital budgeting process is a standard part of financial decision making. This article shows how the certainty equivalent and the risk adjusted discount rate approach convert to one another. Although theoretically the two approaches are different and many studies in the literature argue that the certainty equivalent is superior to the risk adjusted discount rate approach, it is shown the methods are mathematically equivalent if inputs are properly measured. It is shown that if the certainty equivalents of the cash flows are known, the corresponding implied risk adjusted discount rate is computable and rational in application.

INTRODUCTION

In the determination of an optimal cost of capital, certainty equivalence (CE) is theoretically superior to risk adjusted discounted rates (RADR) in common valuation situations, however RADR predominates due to ease of use. Keown, Martin, and Petty (2016) comment the reason that RADR is more popular than certainty equivalent risk adjustment is "purely and simply its ease of implementation". If the theoretically superior certainty equivalent model can be used to develop an implied risk adjusted discount rate, the elements of time and risk would be clearly distinguishable. Such an implied rate could vary across time and according to projected certainty equivalents could provide a superior cost of capital measure. The goal of this article is to review the basis of each methodology and develop an implied risk-adjusted certainty equivalent discount rate to assist in capital investment decisions.

Emphasis on the separation of risk and time is emphasized in Andreoni and Sprenger (2012) and Miao and Zhong (2015) who use utility theory to measure the two components. The present is known while the future is inherently risky. This is problematic when studying time preferences since uncontrolled risk can generate apparently present-biased behavior. Damodaran (2005) states the cost of capital is the main input in which analysts adjust for risk. Since this variable is sensitive to risk and the market risk premium reflects non-diversified risk, the payoff for risk management is hard to trace. It is therefore arguably important to separate the risk and time components.

The RADR method *simultaneously* adjusts for time and risk whereas the certainty equivalent method *separates* the two (Beedles and Joy, 1982). When there is a simultaneous adjustment for time and risk, there is interplay between the two that is not consistent with economics principles. The time and risk adjustment embedded in the RADR figure will be affected differently when N varies. There is no economic rationale for these effects. For example, if the RADR is assumed to be 10% of which 4% is the risk free rate and 6% is the adjustment for risk.

The risk adjustment has a largely different effect depending on the number of periods the cash flow is from the present. In large, multi-year capital investment projects, the risk adjustment using the RADR is arguably not an accurate representation of the risk adjustment. The certainty equivalent method provides a method to fix this problem be extricating time and risk into two separate components.

Gitman and Zutter (2015), Sick (1986), Ben-Tal and Teboulle (2007) and Megginson (1997) agree CE is theoretically superior to RADR. Both note the popularity of RADR stems from two main issues: acceptance by financial decision makers and ease of estimation and application. Firms like to develop several risk classes and then force all projects into one of these classes. This is inconsequential if all errors cancel out across projects and estimation bias is limited which is a reasonable assumption in a large normally distributed sample, but not a singular risky project. Importantly, financial managers should be aware of the large changes in the implied interest rate that result from significant decreases in the certainty equivalent, especially over shorter investment horizons.

This article illustrates why certainty equivalence separates the time and risk components in cost of capital calculations. Furthermore, we illustrate how an implied risk adjusted discount rate may be used to discount risky cash flows of different certainty equivalents. The article proceeds as follows. We first present the background of certainty equivalence and risk adjusted discount rates, then the theoretical development and illustrative examples, followed by the development of the implied risk adjusted rate that allows for different certainty equivalent proportions and risk free discounting for capital investment decisions.

CERTAINTY EQUIVALENCE & RISK ADJUSTED DISCOUNT RATES

In examining a capital budgeting decision using certainty equivalents, the cash flow (in the numerator of the present value calculation) is adjusted to reflect the risk of the cash flow. Once this risk adjustment is made, the cash flow is discounted at the risk free rate to reflect time differentials. This methodology appropriately separates the time and risk factors, allowing for linear adjustments for risk. The certainty equivalent is the value of a certain prospect that yields the same level of utility as the expected utility of an uncertain prospect. For the risk averse investor, this value will always be lower than the expected value of a risky positive cash flow. On the other hand, RADR methodology does not adjust the cash flow in the numerator, but rather adjusts the discount rate in the denominator. The implicit assumption in RADR is that risk increases as time increases as developed in Harris and Pringle (1985).

The debate about appropriate risk adjustment is not new in the finance literature. Robichek and Myers (1966) discussed the problems associated with RADR and since that time, there have been numerous studies published that address the difficulties of application of RADR. Fama (1977) discussed the valuation of multi-period cash flows. Brealey and Myers (2015) state the use of a single risk-adjusted discount rate for long-lived assets will not work when there are multiple phases of project design, in essence presenting a binomial model. If market risk were to change over the life of the project, RADR will not accurately depict the new level of risk. Lewellen (1977, 1979) argued that risky outflows require higher RADR's, while Celec and Pettway (1979) and Hartl (1990) argue the opposite. Berry and Dyson (1980, 1983) and Booth (1982, 1983) continue this debate. Beedles (1978 a, b) suggests that certainty equivalents are superior for estimating the present value of risky cash outflows and Miles and Choi (1979) debate his conclusion. Gallagher and Zumwalt (1991) illustrate how large negative discount rates applied to risky cash outflows may lead an unbounded present value and sensitivity to the number of time periods. We present an application of certainty equivalents that appropriately adjust for risk and provide an implied risk adjusted discount rate which separates the dimensions of time and risk.

Implications of RADR application are numerous. First, there is not a clear sign in the literature as to the direction of adjustment for risky cash outflows. While there is a general consensus that an upward adjustment is appropriate for risky cash inflows, there is not general agreement in the literature as to handle risky cash outflows. Most corporate finance texts advocate lower RADRs for risky cash outflows. Second, the application of a risk adjustment with RADRs is highly arbitrary; generally a 2-4 percentage point adjustment. We show this adjustment is not nearly enough if the certainly equivalent is below 0.95. We advocate that managers apply certainty equivalents in order to more fully grasp the true risk of an investment and to adequately separate the risk and timing components. Such an application could potentially have limited exposure to some of the turmoil in the technology sector; an industry with arguably low certainty equivalents.

Utility functions are not an issue with certainty equivalents or RADR. The necessary variables include the end of period cash payoff, the quantifiable amount of risk, the risk free rate of interest and the price of risk as determined by the market. Since classes of individuals comprise the market, the composite of those classes can quantify individual risk classes. The separation theorem allows for separation of calculation from attitudes toward risk. If there were guaranteed to be an efficient secondary market, only one-period analysis would be necessary. There are many instances in which the secondary market is not efficient on an period by period basis, especially with projects that involve large negative cash outflows many years in the future.

The certainty equivalent level of wealth is the amount at which the investor is indifferent between the risky outcome and the risk free outcome. The investor decides what risk free cash flow he would be willing to accept in exchange for a risky cash flow. For example, if an investor has a 1/300 probability of winning a $10 million lottery, the certain equivalent for a risk neutral investor would be $33,333.33, which is the expected value of a fair game. The period before the drawing of the lottery winner, one has the choice to cash out. What amount would he require to cash out and leave the game? The cash out amount is the certainty equivalent. Given the risk averse utility curve, the certainty equivalent might be $20,000 or some similar number, which is considerably below the expected value of $33,333.33. The certainty equivalent of $20,000 is the cash flow of $33,333.33 adjusted for risk. This amount is then adjusted for time value, by discounting at the risk free rate.

The certainty equivalence principle is applied in Benth, Cartea and Kiesel (2008) in pricing forward contracts. Their limited usage in both personal and corporate financial management is blamed on the relative difficulty of application and determination of certainty equivalents for risky cash flows. In the next section, we will show how the certainty equivalent methodology can and should be properly used to arrive at an implied risk adjusted discount rate.

A BRIDGE BETWEEN CERTAINTY EQUIVALENT AND RISK ADJUSTED DISCOUNT RATE

Although there are debates about whether the certainty equivalent (CE) or the risk adjusted discount rate (RADR) approach should be used to value projects, we can convert one approach to the other. Suppose a possible project requires initial investment $\boldsymbol{I_0}$ and will generate expected cash flows, $\boldsymbol{CF_i}$ at time $\boldsymbol{0 \leq i \leq n}$. We can value the project by both the certainty equivalent and the risk adjusted discount rate approach.

We first value the project by the certainty equivalent approach. Let $\boldsymbol{\alpha_i}$ define the certainty equivalent factor of the i[th] expected cash flow, then, the certainty equivalent of the i[th] expected cash flow is $\boldsymbol{CECF_i = \alpha_i CF_i}$. The present value of $\boldsymbol{CECF_i}$ is $\boldsymbol{CECF_i}$ discounted by the risk-free rate, $\boldsymbol{R_f}$. Therefore, the net present value of the project is:

$$NPV_{CE} = \sum_{i=1}^{n} \frac{CECF_i}{\left(1+R_f\right)^i} - I_0 = \sum_{i=1}^{n} \frac{\alpha_i CF_i}{\left(1+R_f\right)^i} - I_0.$$

Here, $\boldsymbol{NPV_{CE}}$ is the net present value of the project calculated by the certainty equivalent approach. Similarly, one can use the risk adjusted discount rate method to value the same project. Let $\boldsymbol{RADR_i}$ define the risk adjusted discount rate for the i[th] expected cash flow, then the present value of the same project can be calculated as:

$$NPV_{RADR} = \sum_{i=1}^{n} \frac{CF_i}{(1+RADR_i)^i} - I_0.$$

Here, $\boldsymbol{NPV_{RADR}}$ is the net present value of the project calculated by the risk adjusted discount rate approach.

To value a project, we need to decide the certainty equivalent factor, $\boldsymbol{\alpha_i, 0 < i \le n}$, or the risk adjusted discount rate, $\boldsymbol{RADR_i, 0 < i \le n}$. The value of a project depends on $\boldsymbol{\alpha_i}$ or $\boldsymbol{RADR_i}$. Thus, one project can have different values using different methods. However, we can always artificially make the present values calculated by the two approaches equal. That is,

$$\sum_{i=1}^{n} \frac{CF_i}{(1+RADR_i)^i} = \sum_{i=1}^{n} \frac{\alpha_i CF_i}{\left(1+R_f\right)^i}.$$

One solution for the above is:

$$\frac{\alpha_i CF_i}{\left(1+R_f\right)^i} = \frac{CF_i}{(1+RADR_i)^i}.$$

This solution also guarantees that the present values of each period calculated by the two different approaches are the same and as such implies that:

$$RADR_i = \frac{1+R_f}{\alpha_i^{1/i}} - 1,$$

$$or, \qquad \alpha_i = \left(\frac{1+R_f}{1+RADR_i}\right)^i$$

which indicates the following properties of the two valuation approaches:

a. The CE factor (α_i) is always greater than zero.

b. When the certainty equivalent factor is a decreasing (increasing) function of time, the corresponding risk adjusted discount rate is an increasing (decreasing) function. That is, if $\boldsymbol{\alpha_{i+1} < \alpha_i}$, then $\boldsymbol{RADR_{i+1} > RADR_i}$.

c. When $\boldsymbol{\alpha_i = 0}$, $\boldsymbol{RADR_i = R_f}$. That is, if the investor is indifferent between certain and uncertain cash flows, the cash flows should be discounted by the risk-free interest rate and no adjustment is necessary.

d. Since expected cash flows are discounted by the $\boldsymbol{RADR_i}$, time and risk are not separated and thus may not compound properly. The $\boldsymbol{CE}$ method allows for the separation of risk and time by placing risk in the numerator and time in the denominator and discounting at the risk free rate. This is the main argument in favor of the certainty equivalent approach.

In reality, constant risk adjusted discount rates are commonly used. In this case, a constant risk adjusted discount rate could be found by solving (using EXCEL Solver, for instance) the following equation:

$$\sum_{i=1}^{n} \frac{CF_i}{(1+CRADR)^i} = \sum_{i=1}^{n} \frac{\alpha_i CF_i}{\left(1+R_f\right)^i}.$$

Figures 1 and 2 illustrate various certainty equivalents and implied interest rates for periodic model, specifically for the risk free rates of 5% and 10%, respectively. The model assumes an expected but uncertain cash flow of $1,000 with CE factors ($\alpha_i$) ranging from .95 to .05. Figure 1 shows a graphical representation of the implied risk adjusted rate using 1, 2, 3, 10, and 20 periods for the 5% risk free rate. As is seen, the implied rate rises nearly exponentially as the number of period increase and the CE factor decrease. The greater degree of risk aversion the lower the certainty equivalent. For illustrative purposes, one set of numerical calculations is presented in Table 1. In these calculations, it is shown how the single period model varies for a risk free rate of 5% with certainty equivalents ranging from 0.95 to 0.05. The implied rate for a 0.95 CE is 10.53%, representing a 5% reduction for risk and a 5% discount rate. The table shows a dramatic increase in implied rates as the CE's decrease. This increase in implied rates is perhaps greater than intuitively expected. It is definitely larger than the common +/- 2 percentage point adjustment used in RADR. With a CE of 0.50, the implied rate soars to 110%. This is neither complex nor difficult, but illustrative of how the implied rate increases dramatically with decreases in CE's. Intuitively pleasing, this also allows for the separation of risk and time. Figure 2 further illustrate how the implied rate is an increasing function of the risk free rate for risk free rates of 10%, respectively. Again, we see implied rates rise exponentially as CE factors decrease and secondly as the number of periods increases. The graphs illustrate that implied interest rates very sensitive to the certainly equivalent, even for mid-term investments of 2, 3, and 10 years. Since investors are generally interested in returns over more than one year, multi-period models are important.

Table 2 is a multi-period illustration. To illustrate the difference between the implied risk-adjusted discount rates ($\boldsymbol{RADR_i}$) and the constant risk-adjusted discount rate ($\boldsymbol{CRADR}$), we suppose the risk free rate is 5%, and the CE factor, $\boldsymbol{\alpha_i = 1 - 0.5 \times year}$, which is a decreasing function of the year. The $\boldsymbol{RADR_i}$ is calculated for each period. The equivalent $\boldsymbol{CRADR}$ is 13.06%. From Table 2, we see that although the NPV is equal, the implied $\boldsymbol{RADR_i}$ approach discounts the long maturity cash flows more heavily than the $\boldsymbol{CRADR}$ method while the $\boldsymbol{CRADR}$ method discounts the short maturity cash flows more heavily than the CE approach. Intuitively, the longer the maturity, the riskier the cash flow.

CONCLUSION

This paper discusses the two most popular discount rate approaches: the certainty equivalent approach and the risk adjusted discount rate approach. We show that although the theoretical means of these two approaches are different, analysts can convert one to the other, and for any project, there is a one to one map between the certainty equivalent factor and the implied risk adjusted discount rate. In separating the components of time and risk, an implied risk adjusted discount rate is determined to properly account for both elements. This offers executives, especially those in industries with lower certainty equivalents such as pharmaceuticals and technology, the opportunity to accurately price time and risk in modeling for an accurate discount rate.

Figure 1: Implied risk adjusted rates using risk-free rate of 5% for 1, 2, 3, 10 and 20 years.

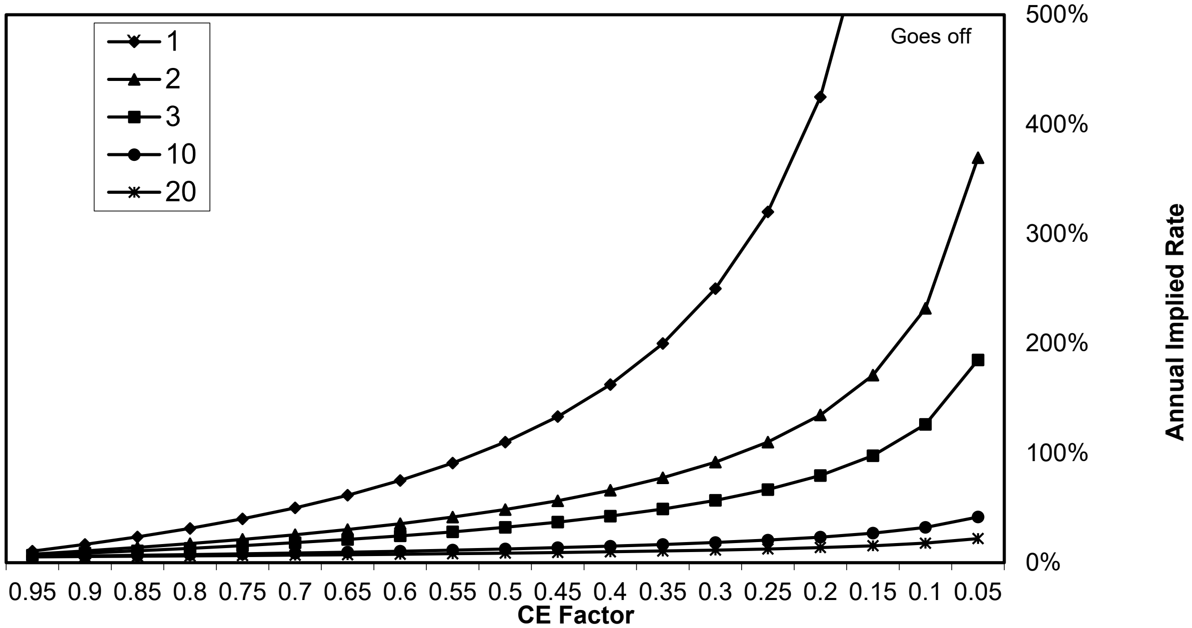

Figure 2: Implied risk adjusted rates using risk-free rates of 10% for 1, 2, 3, 10, and 20 years.

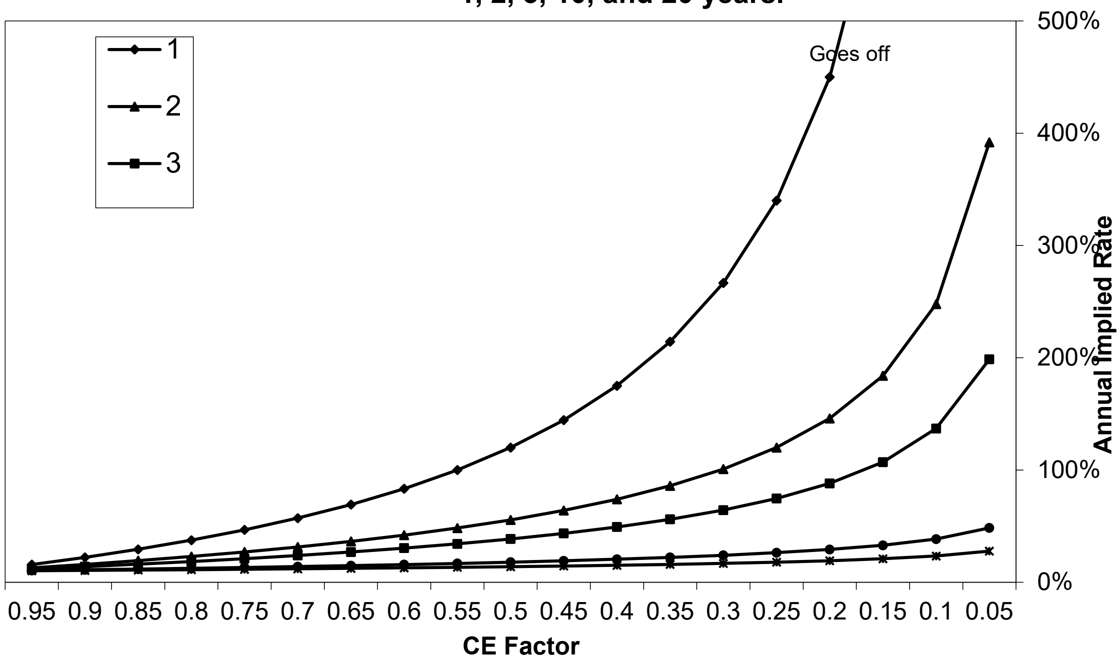

Table 1

IMPLIED RISK ADJUSTED DISCOUNT RATES AT A 5% RISK FREE RATE

This table reports the corresponding risk adjusted discount rates assuming a cash flow of $1,000 with CE factors (α) ranging from .95 to .05. The expected certainty equivalent, ECE, is calculated by multiplying the cash flow by the CE factor (α). The present value of the CE (PV CE) is calculated by dividing the CE by $(1+R_f)$. The implied rate is calculated as follows: $\{[(1+R_f)/\alpha] - 1\}$. The check is calculated as follows: (CE/PV CE). The annuity rate for the 1 period model is the previously calculated implied rate.

Cash Flow	CE-factor	ECE	Risk-free	PV CE	Implied Rate
$1,000	0.95	$950	0.05	$905	10.53%
$1,000	0.90	$900	0.05	$857	16.67%
$1,000	0.85	$850	0.05	$810	23.53%
$1,000	0.80	$800	0.05	$762	31.25%
$1,000	0.75	$750	0.05	$714	40.00%
$1,000	0.70	$700	0.05	$667	50.00%
$1,000	0.65	$650	0.05	$619	61.54%
$1,000	0.60	$600	0.05	$571	75.00%
$1,000	0.55	$550	0.05	$524	90.91%
$1,000	0.50	$500	0.05	$476	110.00%
$1,000	0.45	$450	0.05	$429	133.33%
$1,000	0.40	$400	0.05	$381	162.50%
$1,000	0.35	$350	0.05	$333	200.00%
$1,000	0.30	$300	0.05	$286	250.00%
$1,000	0.25	$250	0.05	$238	320.00%
$1,000	0.20	$200	0.05	$190	425.00%
$1,000	0.15	$150	0.05	$143	600.00%
$1,000	0.10	$100	0.05	$95	950.00%
$1,000	0.05	$50	0.05	$48	2000.00%

Table 2

IMPLIED RISK-ADJUSTED DISCOUNT RATES FOR MULTI PERIODS

For different periods of cash flows, we apply different CE factors and calculate the corresponding risk-adjusted discount rates. PVCECFi is the present value of the certain equivalent cash flows using a 5% risk free rate. PVCFi is the present value of the cash flows calculated by the implied risk-adjusted discount rates. The NPV of those 19 periods cash flow is $6,914.68. The constant risk-adjusted discount rate is CRADR=13.06%. The last column is the present values of the cash flows calculated by the CRADR.

Year(i)	CF_i	$Alpha_i$	$CECF_i$	$PVCECF_i$	$RADR_i$	$PVCF_i$	PV_i
1	1000	0.95	950	904.76	10.53%	904.76	884.51
2	1000	0.9	900	816.33	10.68%	816.33	782.35
3	1000	0.85	850	734.26	10.85%	734.26	691.99
4	1000	0.8	800	658.16	11.02%	658.16	612.07
5	1000	0.75	750	587.64	11.22%	587.64	541.38
6	1000	0.7	700	522.35	11.43%	522.35	478.86
7	1000	0.65	650	461.94	11.66%	461.94	423.55
8	1000	0.6	600	406.10	11.92%	406.10	374.63
9	1000	0.55	550	354.53	12.21%	354.53	331.37
10	1000	0.5	500	306.96	12.54%	306.96	293.10
11	1000	0.45	450	263.11	12.91%	263.11	259.25
12	1000	0.4	400	222.73	13.33%	222.73	229.30
13	1000	0.35	350	185.61	13.83%	185.61	202.82
14	1000	0.3	300	151.52	14.43%	151.52	179.40
15	1000	0.25	250	120.25	15.17%	120.25	158.68
16	1000	0.2	200	91.62	16.11%	91.62	140.35
17	1000	0.15	150	65.44	17.40%	65.44	124.14
18	1000	0.1	100	41.55	19.33%	41.55	109.80
19	1000	0.05	50	19.79	22.93%	19.79	97.12
							$6,914.68

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