BUILDING OPTIMAL RISKY AND UTILITY MAXIMIZING TIAA/CREF PORTFOLIOS

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ABSTRACT

We present a step-by-step process that investors can use to build optimal risky portfolios with Excel Solver and illustrate the process using TIAA/CREF annuities and mutual funds. Because these funds and other tax deferred eligible investments cannot be sold short, investors would face additional challenges in applying the portfolio theory in practice on those products. The inability to short-sell results in an optimal portfolio with low returns and to garner higher returns, investors must select a higher return and find an optimal portfolio for that return.

After forming optimal risky portfolios, we compute the utility for investors with differing levels of risk aversion. We discuss the challenges of applying theory to practice and, the assumptions implicit in forming optimal portfolios, and the limitations of the process. The empirical results shown in this study also help gauge the additional costs in applying this method on investment vehicles that cannot be sold short.

INTRODUCTION

Federal tax law permits taxpayers to invest for retirement using 401(k), 403(b), and/or 457 accounts. These accounts are merely accounts that comply with certain federal requirements and are designed to permit taxpayers to invest for their retirement on a tax sheltered or tax deferred basis. In general, for 2018, eligible taxpayers under the age of 50 are permitted to invest up to $18,500 in a 401(k), 403(b), or 457 account. Taxpayers over the age of 50 are permitted a "catch up" which permits investing an additional $6,000 per year for a total of $24,500 per year in the account.

Typically, employers will select a provider for the retirement account or accounts and the provider will provide support for both the human resource department and employees. The provider and/or the Human Resource Office will provide employees with information regarding the account in general and the features of those accounts. In addition, the specific investment choices permitted under the employees plan are provided. Moreover, employees are provided with resources that provide information about the investment choices such as past returns during specified historical periods (such as year-to-date, one-year, three-year, five-year, ten-year, and since inception), expenses of the various investment options, the types of assets the fund invests in, background of the fund manager, and other fund related information. Representatives of the investment provider may even provide onsite presentations and one-on-one counseling. While, these can provide general investment information, and can highlight the need to diversify your investment portfolio, they fall short of providing a specific investor with a portfolio that will be most rewarding to that specific investor.

The purpose of this paper is to show investors and investment advisors how to create optimal risky and utility maximizing portfolios using Excel. To illustrate the process, we utilize TIAA/CREF investment options and then to use Markowitz (1952) optimization to form an optimal risky portfolio. Once an optimal risky portfolio is formed, we discuss risk aversion and use three levels of risk aversion to form "utility-maximizing" portfolios for investors with each level of risk aversion. Our goal is to provide guidance to investors and/or investment advisors that will permit them to use a simple tool to allocate retirement assets to achieve the highest return with the lowest level of risk.

We selected TIAA/CREF for illustrative purposes because they began offering retirement services to teachers about 100 years ago. Now, TIAA/CREF is a full service financial services company that specializes in serving the needs of academics, researchers, and workers in the medical and cultural fields. As of the first quarter of 2018, TIAA/CREF had nearly $1 trillion in assets under management and was serving 5 million clients in institutional retirement plans. According to Pensions & Investments (2013), TIAA/CREF is one of the largest managers of equity and fixed-income assets (based on assets under management). TIAA/CREF has also received numerous awards for investment performance. For example, Lipper named TIAA/CREF the best overall large fund company based on risk-adjusted performance from 2013-2017 among up to 48 peer companies. Moreover, 67% of TIAA/CREFs funds received an overall Morningstar rating of 4 or 5 stars based on risk-adjusted returns at the end of the first quarter of 2018.

While the above discussion highlights the retirement accounts and mentions sources of information in general, and the importance of TIAA/CREF to some investors in particular, investors remain unaware precisely how assets should be allocated. We fill that void by illustrating how optimal risky portfolios can be formed using Excel Solver. We then form utility maximizing portfolios for investors with differing levels of risk aversion.

The next section discusses risk, return, and the benefit of diversification. It also lists investment options offered by TIAA/CREF. Section 3 discusses our data and the historical returns, variances, and return correlations of TIAA/CREF investment choices. Building optimal risky portfolios using Excel Solver and then forming utility maximizing portfolios is explained and illustrated in Section 4. The paper concludes with the recommended asset allocation based on our dataset and then provides the utility of portfolios for investors with risk aversion scores from one through three. The assumptions, challenges, and limitations of this approach are also discussed.

RISK, RETURN, CORRELATION, AND THE BENEFIT OF DIVERSIFICATION

Finance textbooks often stress that investors should only care about two variables, risk and return (Bodie et al. 2014; Brigham and Ehrhardt 2014; or Smart et al. 2014).

As Equation 1 shows, the return of an investment portfolio is the market value weighted average of the returns of the investments making up the portfolio:

$$E(R_p) = W_a R_a + W_b R_b \tag{1}$$

where R_p is the return on the portfolio; W_a and W_b are the market value weights of the portfolio invested in investments "a" and "b"; and R_a and R_b are the expected returns of investments "a" and "b."

The risk of a portfolio is its variability of returns and can be computed as shown in Equation 2:

$$\sigma_p^2 = W_a^2\sigma_a^2 + W_b^2\sigma_b^2 + 2W_aW_b\sigma_a\sigma_b\rho_{a,b} \tag{2}$$

where σ_p^2 is the variance of the portfolio; W_a^2 and W_b^2 are the squared market value weights for investments "a" and "b"; σ_a^2 and σ_b^2 are the variance of the returns of investments "a" and "b"; σ_a

and σ_b are the standard deviations of the returns of investments "a" and "b"; and $\rho_{a,b}$ is the correlation between investments "a" and "b".

Risk, return, and diversification require further discussion. As illustrated by Equation 2, if the correlation between assets is perfectly positive (+1), there is no benefit to diversification. Conversely, with perfect negative correlation (-1), all risk could be eliminated. In practice, neither of these cases is typically observed. However, if an investor diversifies into an asset class that is not perfectly correlated with the returns of the current portfolio, the risk of the portfolio may be reduced. Therefore, investors should hold a mix of assets that are not highly correlated. As Solnik (1974) shows, both diversifying within a country and between countries is important because of the potential diversification effects.

As portfolio size increases, the portfolio return formula does not change, it remains the market value weighted average of the returns of the investments in the portfolio. However, the formula for portfolio variance changes when portfolio size increases. Besides adding a squared market value weight for the additional investment times the investments variance, more covariance terms are needed for each possible combination of assets. For example, for a portfolio with four investments, Equation 3 shows the corresponding formula.

$$\sigma_p^2 = W_a^2\sigma_a^2 + W_b^2\sigma_b^2 + W_c^2\sigma_c^2 + W_d^2\sigma_d^2 + 2W_aW_b\sigma_a\sigma_b\rho_{a,b} + 2W_aW_c\sigma_a\sigma_c\rho_{a,c} + 2W_aW_d\sigma_a\sigma_d\rho_{a,d} + 2W_bW_c\sigma_b\sigma_c\rho_{b,c} + 2W_bW_d\sigma_b\sigma_d\rho_{b,d} + 2W_cW_d\sigma_c\sigma_d\rho_{c,d} \quad (3)$$

Deciding what asset classes to include in the portfolio and in what proportion is the heart of the portfolio management decision. According to Brinson, Hood, and Beebower (1986) and Brinson, Singer, and Beebower (1991), more than 90 percent of a portfolio's return is due to asset allocation decisions. More recent studies, such as Ibbotson and Kaplan (2000) and Xiong, Ibbotson, Idzorek, and Chen (2010), point out that asset allocation may not be as important in explaining variation in returns across various funds as previously believed. Yet, Ibbotson (2010) concludes asset allocation is still a very important aspect.

Table 1 lists selected TIAA/CREF investments and the name of the investment funds suggests that the assets that some of them hold are dissimilar. Thus, because investment portfolios should take on the risk and return attributes of the underlying asset class, we would expect to have some asset classes with low correlation to other classes. Therefore, it should be possible to build a diversified portfolio from TIAA/CREF annuities or mutual funds. Because investors are only permitted to invest in the investments selected by their employer, we examine two scenarios: annuities only and mutual funds only. The rationale for this choice is that some employers only permit investing in annuities during working years. However, after retirement, the investor may move money as they see fit.

DATA AND METHODOLOGY

Data

The TIAA/CREF investments that we considered are listed in Table 1. Money market investments, targeted retirement funds, and funds with insufficient history to make reliable comparisons were excluded. Daily net asset value (NAV) for the eight variable annuities was extracted directly from TIAA/CREF's website and begins on May 1, 1997. Daily returns for the annuities were computed as shown in Equation 4.

$$r_t = (NAV_t/NAV_{t-1}) - 1 \quad (4)$$

Table 1
TIAA/CREF INVESTMENT CHOICES

TIAA/CREF Variable Annuities	**Inception Date**
CREF Equity Index Account QCEQRX	4/29/1994
CREF Global Equities Account QCGLRX	5/1/1992
CREF Growth Account QCGRRX	4/29/1994
CREF Stock Account QCSTRX	7/31/1952
TIAA Real Estate Account QREARX	10/2/1995
CREF Bond Market Account QCBMRX	3/1/1990
CREF Inflation-Linked Bond Account QCILRX	5/1/1997
CREF Social Choice Account QCSCRX	3/1/1990
TIAA/CREF Mutual Funds	**Inception Date**
TIAA-CREF Equity Index Fund (Retirement) TIQRX	3/31/2006
TIAA-CREF Inflation-Linked Bond Fund (Retirement) TIKRX	3/31/2006
TIAA-CREF International Equity Fund (Retirement) TRERX	10/1/2002
TIAA-CREF International Equity Index Fund (Retirement) TRIEX	10/1/2002
TIAA-CREF Large-Cap Growth Index Fund (Retirement) TRIRX	10/1/2002
TIAA-CREF Large-Cap Value Fund (Retirement) TRLCX	10/1/2002
TIAA-CREF Large-Cap Value Index Fund (Retirement) TRCVX	10/1/2002
TIAA-CREF Mid-Cap Growth Fund (Retirement) TRGMX	10/1/2002
TIAA-CREF Mid-Cap Value Fund (Retirement) TRVRX	10/1/2002
TIAA-CREF S&P 500 Index Fund (Retirement) TRSPX	10/1/2002
TIAA-CREF Small-Cap Blend Index Fund (Retirement) TRBIX	10/1/2002
TIAA-CREF Small-Cap Equity Fund (Retirement) TRSEX	10/1/2002
TIAA-CREF Social Choice Equity Fund (Retirement) TRSCX	10/1/2002

Returns for the 13 mutual funds were extracted from the Center of Research in Securities Prices (CRSP) survivorship bias free mutual fund data base and begin on April 3, 2006. Data for all series end on December 31, 2014.

Correlations of TIAA/CREF investments

Creating a correlation matrix in Excel is a simple process once the analysis tool pack is installed. Simply click on the data tab, and then click on the analysis tab. This will cause a drop-down list box to appear. Select correlation and then select all the cells that contain return data for the selected investments.

Tables 2 and 3 summarize the historical correlations TIAA/CREF variable annuities and

Table 2
CORRELATION OF TIAA/CREF VARIABLE ANNUITIES
Correlations shown in this table are for daily return data from May 2, 1997 through December 31, 2014.

	QCEQRX	QCBMRX	QCGLRX	QCGRRX	QREARX	QCILRX	QCSCRX	QCSTRX
QCEQRX	1							
QCBMRX	-0.221	1						
QCGLRX	0.928	-0.210	1					
QCGRRX	0.963	-0.219	0.889	1				
QREARX	0.190	-0.030	0.191	0.175	1			
QCILRX	-0.201	0.739	-0.184	-0.189	-0.001	1		
QCSCRX	0.984	-0.087	0.919	0.941	0.190	-0.099	1	
QCSTRX	0.988	-0.218	0.970	0.947	0.195	-0.195	0.978	1

Table 3
CORRELATION OF TIAA/CREF MUTUAL FUNDS
Correlations are for daily return data from April 3, 2006 through December 31, 2014.

	TIKRX	TIQRX	TRBIX	TRCVX	TRERX	TRGMX	TRIEX	TRIRX	TRLCX	TRSCX	TRSEX	TRSPX	TRVRX
TIKRX	1												
TIQRX	-0.235	1											
TRBIX	-0.220	0.946	1										
TRCVX	-0.230	0.988	0.922	1									
TRERX	-0.156	0.868	0.803	0.856	1								
TRGMX	-0.222	0.959	0.937	0.921	0.851	1							
TRIEX	-0.149	0.887	0.809	0.877	0.972	0.859	1						
TRIRX	-0.236	0.986	0.926	0.954	0.864	0.970	0.883	1					
TRLCX	-0.225	0.987	0.928	0.994	0.863	0.930	0.881	0.957	1				
TRSCX	-0.235	0.997	0.944	0.986	0.865	0.958	0.884	0.985	0.985	1			
TRSEX	-0.222	0.947	0.994	0.921	0.805	0.941	0.811	0.929	0.927	0.944	1		
TRSPX	-0.238	0.997	0.927	0.990	0.868	0.947	0.890	0.985	0.987	0.995	0.927	1	
TRVRX	-0.213	0.985	0.950	0.979	0.863	0.955	0.876	0.964	0.983	0.986	0.950	0.978	1

mutual funds, respectively. Annuity correlations reveal that some asset combinations are highly correlated and would not offer much diversification benefit. Thus, an investor might hold only one of those assets because they can be viewed as compliments. For example, the CREF Equity Index Account (QCEQRX) and the CREF Stock Account (QCSTRX) have a correlation coefficient of 0.988. However, other asset combinations such as the CREF Equity Index Account (QCEQRX) and the TIAA Real Estate Account (QREARX) have a small positive correlation coefficient of 0.190 while the CREF Equity Index Account (QCEQRX) and the CREF Bond Market Account QCBMRX have a negative correlation (-0.221). Both of these combinations potentially offer tremendous diversification potential.

Results of mutual fund correlations are similar to those of annuities. Some asset combinations are highly correlated and would not offer much diversification benefit such as the TIAA-CREF Equity Index Fund (Retirement) TIQRX and the TIAA-CREF Social ChoiceEquity Fund (Retirement) TRSCX at 0.997. However, other asset combinations potentially offer tremendous diversification potential (the TIAA-CREF Inflation-Linked Bond Fund (Retirement) TIKRX and the S&P 500 Index Fund (Retirement) TRSPX at -0.238.

OPTIMAL RISKY PORTFOLIOS AND UTILITY MAXIMIZING PORTFOLIOS

Optimal risky variable annuity portfolios

Harry Markowitz's (1952) Nobel Prize winning research created Modern Portfolio Theory which asserts that investors should make investment decisions using the mean, variance, and covariance (or correlation) of securities, and this concept is widely accepted in the investment industry. The optimization of risky portfolios focuses on two aspects: maximizing returns while holding risk constant or minimizing risk while maintaining the same level of return. The goal of portfolio optimization is to maximize portfolio return per unit of risk. With a risk-free asset, this can be simplified to maximizing the Sharpe ratio of a portfolio, which is its excess return per unit of total portfolio risk. Equation 5 illustrates maximizing the Sharpe ratio.

Pure theory suggests that an optimal portfolio can be found by correctly combining assets and the optimal portfolio will dominate all other portfolios in terms of risk and return. Once this dominant portfolio is found, it can be combined with a risk-free asset to form the Capital Market Line (CML). Portfolios on the CML will dominate all others in risk-return space. All of these portfolios will have the same excess return per unit of risk but their excess return per unit of risk will be higher than any other portfolio.

Maximizing the Sharpe ratio is tricky with a mutual fund or annuity investment because neither the investments nor the risk-free asset can be sold short. The following maximization problem shown in Equation 5 defines the optimization of a mutual fund or annuity portfolio:

$$Max\ \vartheta = \frac{E(r_w - C)}{\sigma_w}$$

$$max\vartheta = \frac{E(r_w) - c}{\sigma_w} \quad (5)$$

given the relationships in Equations 6 through Equation 11.

$$\sum_{i=0}^{n} w_i = 1 \quad \sum_{i=1}^{N} w_i = 1 \quad w_i \geq 0, i = 1,2,\dots,N \quad (6)$$

where

$$E(r_w) = w^T \times R = \sum_{i=1}^{N} w_i E(r_i) \quad (7)$$

$$\sigma_w = \sqrt{\sum_{i=0}^{n}\sum_{j=0}^{n} w_i w_j \sigma_{ij}} \quad E(r_w) = W^T \times R = \sum_{i=1}^{N} w_i E(r_i) \quad (8)$$

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \quad (9)$$

$$R = \begin{bmatrix} E(r_1) \\ E(r_2) \\ \vdots \\ E(r_N) \end{bmatrix} \quad (10)$$

and

$$S = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_{NN} \end{bmatrix} \quad (11)$$

where w_i is the market value weight invested in investment i; $E(r_i)$ is the expected rate of return of investment i; σ_{ij} is the covariance between investment i and investment j; and c is a constant. Changing c permits finding infinite combinations of w_i and therefore creating the efficient frontier with the selected mutual fund or annuity investments. These portfolios dominate all other choices in terms of return for a given level of risk. If the risk-free rate is used for c, a theoretical optimal risky portfolio may be found by solving the problem above. To better illustrate the application of portfolio optimization in practice, an example is provided using Microsoft Excel with TIAA/CREF annuities. Assume that an investor has selected eight annuities to consider based on their investment objectives. To construct an optimal portfolio, the investor must compute historical returns over some period such as May 1997 to December 2014 in this example. Table 4 shows the summary statistics for selected annuities.

Table 4 SUMMARY STATISTICS FOR TIAA/CREF VARIABLE ANNUITY RETURNS								
Daily	QCEQRX	QCBMRX	QCGLRX	QCGRRX	QREARX	QCILRX	QCSCRX	QCSTRX
Mean	0.0004	0.0002	0.0003	0.0003	0.0002	0.0002	0.0003	0.0003
Std	0.0128	0.0023	0.0119	0.0140	0.0013	0.0035	0.0074	0.0122
Var	0.0002	0.0000	0.0001	0.0002	0.0000	0.0000	0.0001	0.0001
Annual	QCEQRX	QCBMRX	QCGLRX	QCGRRX	QREARX	QCILRX	QCSCRX	QCSTRX
Mean	0.0919	0.0532	0.0735	0.0811	0.0613	0.0556	0.0709	0.0839
Std	0.2027	0.0369	0.1885	0.2222	0.0207	0.0549	0.1174	0.1936
Var	0.0411	0.0014	0.0355	0.0494	0.0004	0.0030	0.0138	0.0375

Computing return metrics is straightforward. Daily average returns are computed by entering the following into the cell:
=AVERAGE(first cell in return column:last cell in return column) and then pressing enter. Standard deviation is computed by entering: =STDEV(first cell in return column:last cell in return column) and then pressing enter. Variance is computed by entering: =VAR(first cell in return column:last cell in return column) and then pressing enter.

Converting from daily to annual returns and variances can easily be accommodated by multiplying the daily return or daily variance cells by 252 (the approximate number of trading days in a year). Annual standard deviation can be computed by taking the square root of annual variance or multiplying daily standard deviation by the square root of 252.

At this point, the investor needs to set up to solve the constrained optimal problem using Equation 5. This task can be accomplished in Excel using the Solver tool. The process begins by first setting up Excel. In addition to needing the returns, standard deviations, and variances above, a covariance matrix must be created. We created a correlation matrix earlier by selecting the data tab in Excel, then data analysis, and then selecting the input range (the cells containing the daily returns of the investments of interest). The Excel output was a triangle (lower left) of the correlation of each combination of assets. The complete correlation matrix can be created by copying the lower row and then using the transpose function in paste special to paste those values into the last column. That process is repeated until all cells have a value (note that the diagonal will be one).

To create a covariance matrix, it is convenient to copy the average returns, standard deviations and variances, from the investments and pate them in column format, and also paste them using paste special and transpose to present them in row format. It is also convenient to paste the full correlation matrix nearby. Once that is completed, the complete covariance matrix can be constructed. Starting at the upper left-hand cell of the covariance matrix, enter the cell reference for the standard deviation for that investment (from the column data) times the cell reference for the standard deviation for that investment (from the row data) times the cell reference for the correlation of that asset with itself. The formula in that cell can be copied and pasted to the other cells in the covariance matrix. Some changes in the cells will need to be made, and some changes can be minimized by using the $ command to lock cell references.

The final step of setting up the Excel template is to make a column that lists each investment and then the words total, average, standard deviation, variance, and Sharpe ratio (as in

Table 5). The next column will be titled weights. This will serve as the template for the solver output (including the formulas for return and variance).

Once the spreadsheet is set up, the process begins by forming an arbitrary portfolio. For example, a portfolio equally split among the eight target funds. The portfolio mean and standard deviation (σ) may be computed using Equations 7 and 8, respectively.

This is accomplished in Excel by entering the following equations. To compute portfolio return in a way that solver can update it when it runs, enter the following equation in the portfolio return output cell:

=MMULT(TRANSPOSE(begining cell in portfolio weight range:ending cell in portfolio weight range), begining cell in asset return range:ending cell in asset return range)

but do not press Enter! To enter a formula that solver can iterate, press and hold Ctrl, Shift, and then Enter.

To compute portfolio variance in a way that solver can update it when it runs, enter the following equation in the portfolio return output cell:

=MMULT(MMULT(TRANSPOSE(begining cell in portfolio weight range:ending cell in portfolio weight range),begining cell covariance matrix:ending cell in covariance matrix), begining cell in portfolio weight range:ending cell in portfolio weight range)

and again do not press Enter at this point. To enter a formula that solver can iterate, press and hold Ctrl, Shift, and then Enter.

In the standard deviation output cell, enter:

=SQRT(varaice cell reference).

For the Sharpe ratio output cell enter:

=(portfolio return cell reference-risk-free rate cell reference / portfolio standard deviation cell reference).

The goal of the optimization is to maximize the Sharpe ratio of the portfolio as shown in Equation 5. Therefore, the ratio is computed so that the optimal solution can be derived in the next step. The risk-free rate in this case is assumed to be 3 percent; however, this can easily be changed and the scenario re-run to ascertain the impact of the choice of risk-free rate on the optimal portfolio.

The Solver function in Excel can find the maximum, minimum, or a specified number in a specific cell by changing parameters. The parameters are the cells containing the investment weights in each of the eight selected investments. Two constraints must be added to the Solver to further limit solutions. The first constraint is that the cells containing the weight in each investment must be ≥ 0 (no short sales). Secondly, the cell containing the sum of the weights must be 1 or 100 percent.

Table 5 reveals that there is a solution to the optimization problem. The optimal portfolio is 78.15% TIAA Real Estate Account (QREARX), 20.90% CREF Bond Market Account (QCBMRX), and 0.95% CREF Equity Index Account (QCEQRX). The optimal risky portfolio has an expected return of 5.99% and a Sharpe measure of 1.6616. In practice, as opposed to pure theory, an investor can't short sell the riskless asset to create a portfolio with a higher return. Thus, if an investor desires a higher rate of return, they must select a portfolio that is mean-variance

Table 5
OPTIMAL TIAA/CREF ANNUITY PORTFOLIOS

Portfolio	A	B	C	D	E	F
Target	Optimal	7%	7.50%	8%	8.50%	Max return
FUND	**Weight**	**Weight**	**Weight**	**Weight**	**Weight**	**Weight**
QCEQRX	0.0095	0.2844	0.4480	0.6116	0.7751	1
QCBMRX	0.2090	0	0	0	0	0
QCGLRX	0	0	0	0	0	0
QCGRRX	0	0	0	0	0	0
QREARX	0.7815	0.7156	0.5520	0.3884	0.2249	0
QCILRX	0	0	0	0	0	0
QCSCRX	0	0	0	0	0	0
QCSTRX	0	0	0	0	0	0
Total	1	1	1	1	1	1
Portfolio	A	B	C	D	E	F
Average	0.0599	0.0700	0.0750	0.0800	0.0850	0.0919
Var	0.0003	0.0039	0.0088	0.0158	0.0250	0.0411
Std	0.0180	0.0622	0.0937	0.1257	0.1581	0.2027
Sharpe Ratio	1.6616	0.6431	0.4805	0.3977	0.3480	0.3053

inefficient. To accommodate investors with differential return preferences, we used solver to solve for optimal risky portfolios with a range of different levels of return. This is accomplished in solver by adding a third constraint requiring that the portfolio return output cell equal a specified value and then re-running solver to obtain the optimal portfolio for that level of return. As an example, we repeated this process for desired return levels of 7%, 7.5%, 8%, 8.5%, 9%, and 9.18%. We will discuss these portfolios in greater detail once we select the utility maximizing portfolios.

Optimal risky mutual fund portfolios

To find optimal risky mutual fund portfolios, the process used for variable annuities can be repeated. Table 6 shows the summary statistics for selected annuities.

Table 6
SUMMARY STATISTICS FOR TIAA/CREF MUTUAL FUND RETURNS

Date	TIKRX	TIQRX	TRBIX	TRCVX	TRERX	TRGMX	TRIEX	TRIRX	TRLCX	TRSCX	TRSEX	TRSPX	TRVRX
4/3/2006	0.0000	0.0010	-0.0079	0.0033	0.0113	-0.0005	0.0080	0.0008	0.0013	0.0009	-0.0061	0.0020	0.0017
4/4/2006	-0.0010	0.0050	0.0043	0.0073	0.0090	0.0044	0.0095	0.0042	0.0066	0.0055	0.0043	0.0061	0.0039
4/5/2006	0.0030	0.0050	0.0049	0.0059	0.0022	0.0065	0.0016	0.0033	0.0046	0.0037	0.0049	0.0047	0.0061
4/6/2006	-0.0030	-0.0020	0.0000	-0.0033	0.0037	0.0027	0.0047	-0.0008	-0.0013	-0.0018	0.0000	-0.0020	-0.0011
12/26/2014	0.0009	0.0038	0.0069	0.0022	0.0027	0.0029	0.0028	0.0047	0.0028	0.0035	0.0065	0.0035	0.0029
12/29/2014	0.0000	0.0013	0.0037	0.0022	-0.0054	0.0010	-0.0022	0.0005	0.0017	0.0012	0.0029	0.0009	0.0049
12/30/2014	0.0009	-0.0050	-0.0047	-0.0033	-0.0099	-0.0057	-0.0105	-0.0056	-0.0039	-0.0052	-0.0047	-0.0047	-0.0045
12/31/2014	0.0018	-0.0094	-0.0069	-0.0110	-0.0036	-0.0053	-0.0050	-0.0090	-0.0089	-0.0093	-0.0076	-0.0104	-0.0095
Daily	TIKRX	TIQRX	TRBIX	TRCVX	TRERX	TRGMX	TRIEX	TRIRX	TRLCX	TRSCX	TRSEX	TRSPX	TRVRX
Average	0.0002	0.0004	0.0004	0.0004	0.0002	0.0004	0.0002	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
Std	0.0040	0.0138	0.0173	0.0145	0.0155	0.0153	0.0147	0.0130	0.0152	0.0138	0.0171	0.0135	0.0149
Var	0.0000	0.0002	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002
Annual	TIKRX	TIQRX	TRBIX	TRCVX	TRERX	TRGMX	TRIEX	TRIRX	TRLCX	TRSCX	TRSEX	TRSPX	TRVRX
Average	0.0452	0.0961	0.1018	0.0899	0.0541	0.1031	0.0504	0.1018	0.0905	0.0944	0.0977	0.0942	0.1057
Std	0.0641	0.2191	0.2745	0.2308	0.2460	0.2428	0.2341	0.2058	0.2417	0.2185	0.2715	0.2141	0.2369
Var	0.0041	0.0480	0.0754	0.0533	0.0605	0.0589	0.0548	0.0423	0.0584	0.0477	0.0737	0.0459	0.0561

Table 7 reveals that there is a solution to the optimization. The optimal portfolio is 71.64% TIAA-CREF Inflation-Linked Bond Fund (Retirement) TIKRX and 28.36% TIAA-CREF Large-Cap Growth Index Fund (Retirement) TRIRX. The optimal risky portfolio has an expected return of 6.12% and a Sharpe measure of 0.4792. Because an investor can't short sell the riskless asset, if the investor desires a higher rate of return they must select a portfolio that is mean-variance inefficient. To accommodate investors with differential return preferences, we used solver to solve for optimal risky portfolios with desired levels of return of 7%, 7.5%, 8%, 8.5%, 9%, 9.5% , 10%, and 10.5%.

Assessing risk aversion and utility

Investors need to choose among competing combinations and should do so considering their own risk tolerance. While an investor could be risk-averse, risk-neutral, or risk-loving, a common assumption is that most investors are *risk-averse.* A *risk-averse investor* is simply one who dislikes uncertainties or assuming risk (i.e., prefers less risk to more risk for a given level of return). The optimal portfolios have the highest expected returns given the degree of risk or lowest degree of risk given the level of return. Choosing among competing optimal portfolios is a risk and return trade-off. Thus, the choice depends on the investors' risk tolerance.

Table 7
OPTIMAL TIAA/CREF MUTUAL FUND PORTFOLIOS

Portfolio	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Target	Optimal	7%	7.50%	8%	8.50%	9%	9.5%	10%	10.50%
FUND	Weight	Weight	Weight	Weight	Weight	Weight	Weight	Weight	Weight
TIKRX	0.7164	0.5619	0.4737	0.3855	0.2972	0.2090	0.1208	0.0326	0
TIQRX	0	0	0	0	0	0	0	0	0
TRBIX	0	0	0	0	0	0	0	0	0
TRCVX	0	0	0	0	0	0	0	0	0
TRERX	0	0	0	0	0	0	0	0	0
TRGMX	0	0	0	0	0	0	0	0	0
TRIEX	0	0	0	0	0	0	0	0	0
TRIRX	0.2836	0.4381	0.5263	0.6145	0.7028	0.7910	0.8792	0.9674	0.1788
TRLCX	0	0	0	0	0	0	0	0	0
TRSCX	0	0	0	0	0	0	0	0	0
TRSEX	0	0	0	0	0	0	0	0	0
TRSPX	0	0	0	0	0	0	0	0	0
TRVRX	0	0	0	0	0	0	0	0	0.8212
Total	1	1	1	1	1	1	1	1	1
Portfolio	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Average	0.0612	0.0700	0.0750	0.0800	0.0850	0.0900	0.0950	0.1000	0.1050
Var	0.0043	0.0079	0.0111	0.0151	0.0200	0.0256	0.0321	0.0394	0.0530
Std	0.0652	0.0888	0.1054	0.1230	0.1413	0.1601	0.1792	0.1986	0.2302
Sharpe Ratio	0.4792	0.4503	0.4271	0.4065	0.3892	0.3747	0.3626	0.3525	0.3257

Risk and risk aversion are used to decide how to allocate wealth among competing investment opportunities. Investors hold different portfolios due to their differing attitudes toward risk.

Examining how to choose among competing alternatives is important while maximizing the investor's satisfaction. Thus, the goal is to maximize the investor's utility. Equation 12 is a commonly used utility function based on an investor's investment outcome:

$$U = E(r) - \frac{1}{2}A\sigma^2 \quad (12)$$

where *U* is the investor's utility; *E(r)* is the expected return of the portfolio; ½ is a constant scaling factor; "*A*" is the investor's risk tolerance or risk aversion score; and σ^2 is the portfolio variance. This formula reveals that utility changes are intuitive. An investor prefers to have a higher expected return, but feels penalized to bear a higher degree of risk, as measured by the portfolio variance. As the expected return of a portfolio increases, so does the investor's utility, ceteris paribus. An investor's utility also decreases as risk increases. However, the decrease depends on the investor's risk aversion score "*A*". Some investors place a large penalty on a portfolio for an increase in risk, as represented by a higher "A", while other investors place much less of a penalty for a risk increase. More than one portfolio could be equally satisfying for an investor.

Creating utility maximizing portfolios

The optimal risky portfolios derived in the previous section do not account for the investor's risk preference. Although the portfolios are optimized based on Markowitz's mean-variance analysis, the ultimate choice still depends on the investor's risk attitude. The mutual fund separation theorem (Cass and Stiglitz 1970; Ross 1978; Chamberlain 1983) states that investors who are making optimal investment choices between a set of risky assets and a risk-free security should all hold the same portfolio of risky assets and their risk attitude does not influence the relative proportion of funds invested across different risky assets. Thus, the risk-preference-adjusted optimization does not need to re-create the optimal weights among risky assets. It simply needs to find the appropriate weights for the risk-free asset and the optimal risky portfolio. An optimal risky portfolio is created based on objective information including the expected risk and return, and a utility maximizing portfolio mixes the optimal risky portfolio with the risk-free asset and is based on the investor's subjective risk preference.

In Theory, the task is to quantify an investor's risk preference, which is typically done with a utility function. The previous section presented a common utility function. Therefore, Equation 13 shows an objective function:

$$\max U = E(r_p) - \frac{1}{2}A\sigma_p^2 \tag{13}$$

where r_p is the portfolio's expected rate of return and σ^2_p is the portfolio's expected variance. An investor allocates capital between the optimal risky portfolio and risk-free asset. Assume that the weight invested in the optimal risky portfolio is x. Thus, Equations 14 and 15 describe the expected rate of return $E(r_p)$ and expected variance σ^2_p for the portfolio, respectively:

$$E(r_p) = xE(r_w) + (1-x)r_f = r_f + x\big(E(r_w) - r_f\big) \tag{14}$$

$$\sigma_p^2 = x^2\sigma_w^2 \tag{15}$$

The target function of the maximization problem becomes Equation 16:

$$\max U = r_f + x\big(E(r_w) - r_f\big) - {}^1\!/_2\, Ax^2\sigma_w^2 \tag{16}$$

To find the optimal weight (x) that is needed to maximize an investor's utility, the first order derivative of the expression regarding x should be set at zero as shown in Equation 17. By doing so, an optimal weight (x) may be computed in Equation 18:

$$\frac{dU}{dx} = \big(E(r_w) - r_f\big) - Ax\sigma_w^2 = 0 \tag{17}$$

$$x^* = \frac{E(r_w) - r_f}{A\sigma_w^2} \tag{18}$$

However, the practice of building optimal mutual fund or variable annuity portfolios in practice differs from pure theory and portfolio choices may be mean variance inefficient, as we have shown in Table 5 and 7. This arises due to the inability to short-sell the risky assets (mutual funds or variable annuities) and the inability to short-sell the risk-free asset. Thus, investors seeking higher return must select higher risk but less efficient portfolios. Utility maximization is troublesome because portfolio excess return per unit of risk is not constant because of the inability to short sell the risk-free asset. Despite this setback, we can approximate and investors utility by using Equation 12 and tabulating utility results for our optimal portfolios for each return level for investors with differing risk aversion levels. Table 8 and 9 provide the utility of selected variable annuity and mutual fund portfolios, respectively.

Table 8
UTILITY MAXIMIZING TIAA/CREF ANNUITY PORTFOLIOS

Risk Aversion Score	Portfolios					
	A	B	C	D	E	F
1	0.0597	0.0681	0.0706	0.0721	0.0725	0.0713
2	0.0596	0.0661	0.0662	0.0642	0.0600	0.0508
3	0.0594	0.0642	0.0618	0.0563	0.0475	0.0303

As shown, investors with different risk attitudes will desire different portfolios. While the optimal portfolio is 78.15% TIAA Real Estate Account (QREARX), 20.90% CREF Bond Market Account (QCBMRX), and 0.95% CREF Equity Index Account (QCEQRX), it was not the utility maximizing portfolio for any level of risk aversion that we used. Investors that are not sensitive to risk will prefer portfolio E. Table 5 shows that portfolio E would have 22.49% in the TIAA Real Estate Account (QREARX) and 77.51% in the CREF Equity Index Account (QCEQRX). It has an expected return of 8.5%, an expected standard deviation of 15.8%, and a Sharpe measure of 0.3479. The most risk averse investors in our example will prefer portfolio B that has 28.44% in the TIAA Real Estate Account (QREARX) and 71.56% in the CREF Equity Index Account (QCEQRX). The expected return is 7.5%, the expected standard deviation is 9.4%, and the Sharpe measure is 0.6431.

It is worthy to note that pure theory would create a CML with a linear risk-return tradeoff and all efficient portfolios would share the same Sharpe measure. However, pure theory and practice collide because of short sale constraints on investments and the risk-free asset. While investors can opt for higher returns than the optimal risky portfolio delivers, the cost of doing so is a decreasing Sharpe measure.

Table 9
UTILITY MAXIMIZING TIAA/CREF MUTUAL FUND PORTFOLIOS

Risk Aversion Score	A	B	C	D	E	F	G	H	I
2	0.0591	0.0661	0.0695	0.0724	0.0750	0.0772	0.0789	0.0803	0.0785
3	0.0570	0.0621	0.0639	0.0649	0.0650	0.0644	0.0629	0.0606	0.0520
1	0.0549	0.0582	0.0584	0.0573	0.0550	0.0515	0.0468	0.0408	0.0255

As with annuities, mutual fund investors with different risk attitudes will desire different portfolios. While the optimal portfolio is 71.64% TIAA-CREF Inflation-Linked Bond Fund (Retirement) TIKRX and 28.36% TIAA-CREF Large-Cap Growth Index Fund (Retirement) TRIRX, it is not the utility maximizing portfolio for any of our hypothetical investors. Investors that are not sensitive to risk will prefer portfolio H which is 3.26% TIAA-CREF Inflation-Linked Bond Fund (Retirement) TIKRX and 96.74% TIAA-CREF Large-Cap

Growth Index Fund (Retirement) TRIRX. This portfolio has an expected return of 10%, an expected standard deviation of 19.9%, and a Sharpe measure of .3525. The most risk averse investors in our example will prefer portfolio C which is 47.37% TIAA-CREF Inflation-Linked Bond Fund (Retirement) TIKRX and 52.63% TIAA-CREF Large-Cap Growth Index Fund (Retirement) TRIRX. This portfolio has an expected return of 7.5%, an expected standard deviation of 10.5%, and a Sharpe measure of 0.4271.

CONCLUSION

We review how to compute the risk and return of managed portfolios and illustrate the benefits of diversification. After presenting the theory, we apply the theory to TIAA/CREF data to illustrate how to use the Solver function in Excel. Using the Solver function in Excel provides investors with a step-by-step process to form optimal risky portfolios. After discussing how to form an optimal risky portfolio we address risk aversion and utility as a prelude to forming utility maximizing portfolios.

Using data for TIAA/CREF annuities and mutual funds, we illustrate the process and provide optimal risky and utility maximizing portfolios for select investments during a recent time period. Before an investor implements any of our solutions, one caveat must be clear. An assumption of pure theory is that the historical return data used is represents a good estimate of future returns, variances, and correlations. If this is true, the output should be a good guide to future asset allocation. Unfortunately, in practice, some investments have insufficient time histories to permit making this assumption. Moreover, short time periods can be distorted by major market disturbances as witnessed in the recent financial crisis. A possible solution to this problem is to use indexes as the underlying asset and infer from index allocation the allocation to specific investments that have the index as their benchmark.

A major take-away is that in practice, where short-selling is prohibited, the CML will not be linear. Thus, investors desiring a return greater than that delivered by the optimal portfolio must select an optimal portfolio for a higher return. However, the result will be a less efficient portfolio in terms of excess risk per unit of return.

A restriction to our study is the availability of the data set. Our sample period spans mostly the strong bull market after 1990’s. For investors who wish to follow the procedure to optimize their retirement portfolios, we note that the optimization using data from shorter period should be utilized with caution. For most of investors, the expected investment horizon until retirement may be longer than what our sample period covers. It is worth considering how a major shift in the market regime would play a role in forming investment strategy for retirees.

Our study can be extended in a few ways. First, in response to the limitation specified above, a future study using indexes and their returns may be worth exploring. While a study using index would potentially overlook the effects from the fund providers, the longer available sample period with indexes would allow further research on other issues such as rebalancing. Another potential extension is to study the benefit of optimization within specific fund family. Many tax deferred eligible retirement plans are tied to a specific fund family. Whether or not it is worth to be restricted in investment selection in order to enjoy the tax benefit is one point of interest. Also, employers who sponsor retirement plans may also wish to take further consideration in this regard when choosing the providers.

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