# REVISITING THE SELF-SUSTAINABLE GROWTH RATE

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## ABSTRACT

*The concept of a "Self-Sustainable Rate of Growth", which has its origins in the seminal work of Miller and Modigliani, is widely taught to students of finance as an important tool for long-range planning and forecasting. Although the formulations vary somewhat, most introductory- and intermediate-level Financial Management texts provide a description of the concept, and a simple formula for its estimation.*

*However, these simple formulas are built upon underlying assumptions which are unlikely to hold in practice, and as a result, the estimates they produce are unreliable at best. In this paper, we provide a brief review of the internal- and sustainable-growth-rate formulations presented in popular financial management textbooks. We then demonstrate that the traditional formulas are not reliable when firms have material fixed costs in their cost structures (i.e., whenever the degree of operating leverage is not equal to 1). We provide observational evidence from a sample of US firms that this condition is the rule, rather than the exception.*

*We then address the problems associated with the use of these formulas by describing a more robust alternative procedure for estimation of the IGR and SGR which incorporates information about the subject firm's fixed- and variable costs. In addition, because the details of firms' cost structures are not publicly available, we explore the use of the Degree of Operating Leverage concept as a way of incorporating fixed costs into the IGR and SGR calculations.*

## INTRODUCTION

Although widely cited in popular textbooks, the concept of a self-sustainable growth rate has been, in our experience, something of a stumbling block for students in the intermediate Corporate Finance course. While the concept is intuitively appealing and simple to calculate, students who dive a bit deeper into the financial planning process often discover that the result of the textbook formula does not reconcile with the insights gained from a more detailed model. As we will demonstrate, the popular model contains implicit assumptions that are inconsistent with typical business practices, and consequently the resulting estimates are too imprecise to be of any practical use. The objectives of this paper are to (1) point out the shortcomings of the traditional model of the sustainable growth rate, (2) to demonstrate that these shortcomings lead to material errors when the model is applied to real-world firms, and (3) to propose more robust alternative formulas for the sustainable and internal growth rates, which incorporate information about the firm's structure of fixed and variable costs.

In the following sections, we will provide a brief review of relevant literature, a discussion of the intuition underlying the sustainable growth rate concept, and a demonstration of the popular model's failings. We then offer an alternative formulation for the self-sustainable growth rate which addresses at least one of these shortcomings. In addition, we explore the relationship between the sustainable growth rate and the degree of operating leverage (*DOL*), and provide evidence that the majority of firms do not satisfy the assumption of *DOL*=1 which is implicit in the popular models.

## LITERATURE REVIEW

In their seminal paper on dividend policy, Miller and Modigliani (1961) were the first authors to propose that the rate of growth in a firm's market value could be expressed as a function of the rate of return on its projects and its dividend policy. Babcock (1970) was among the first to identify factors affecting a firm's maximum sustainable rate of growth. Higgins (1977) generalizes and extends the concept of a sustainable growth rate in sales, defining the concept in terms of profitability, asset efficiency, and leverage (in addition to dividend policy). Other authors (for example Bruner 1991, Ashta 2008, and Chen, Gupta, Lee, and Lee (2013)) have produced various refinements and extensions. Arora, Kumar, and Verma (2018) assess the suitability of two different methods of calculating the sustainable growth rate, and, using data from a sample of Indian manufacturing firms, identify the most important factors in determining the SGR.

However, the popular model continues to be standard fare in financial management textbooks; for example, Ross, Westerfield and Jordan (2021), Brigham and Houston (2019), Brealey, Myers and Marcus (2020), and Berk, DeMarzo, and Harford (2021). Of these, Ross, et al provide the most detailed treatment, providing a simple extension that allows the analyst to use contemporaneous observations of net income and shareholders' equity in calculating ROE. Scholars searching the internet for information on the concept will find that sources such as Investopedia (Murphy 2022) and the Corporate Finance Institute (corporatefinanceinstitute.com 2020) also define the concept in its most basic form.

Some of these sources advocate for the usefulness of the concept; for example, according to Murphy (2022), "The SGR is used by businesses to plan long-term growth, capital acquisitions, cash flow projections, and borrowing strategies." None, however, seem to acknowledge the limitations of the popular model. As this note will demonstrate, the popular formulations of SGR lack the precision needed in order to serve as valuable tools for financial planning.

## INTUITION OF THE POPULAR MODEL

The concepts of internal growth rate (IGR) and sustainable growth rate (SGR) are different, but closely related. While SGR is more frequently cited, it seems (anecdotally at least) that students can more readily grasp the intuition behind IGR, which then logically extends to

SGR. Many textbooks (for example, Ross et al. (2021), and Berk et al. (2021)) present the concepts in this order, and we follow that example here.

Ross et al. (2021) define the internal growth rate as "the maximum growth rate that can be achieved with no external financing of any kind". That is, by what percentage can the firm's assets grow, using only capital from retained earnings? This is easily represented as the year-over-year increase in total assets divided by the beginning value:

$$IGR = \frac{ASSETS_1 - ASSETS_0}{ASSETS_0}$$

With *b* representing the retention ratio, *NI* representing Net Income, and allowing that assets can *only* increase via retained earnings, it is apparent that the increase in assets can be no greater than: $ASSETS_1 - ASSETS_0 = NI_1 \times b$

Substituting this expression into the numerator, we obtain the traditional formulation for IGR:

$$IGR = \frac{NI_1 \times b}{ASSETS_0}$$

A common shortcut is to note that Return on Assets (ROA) is defined as $NI/ASSETS$, and thereby restate this result as

$$IGR = ROA \times b$$

This is the definition for IGR presented in many popular sources, such as Berk et al (2021). This shortcut implies that it is possible to use the easily obtainable value for ROA in the IGR calculation. However, as noted by Ross et al (2021), this is not quite correct, since the original expression calls for the amount of net income earned during the observation period, and the level of assets observed at the *beginning* of that period. This would not be consistent with the usual definition of ROA, which compares net income to the end-of-period level of assets. Ross et al (2021) and others provide a definition of IGR which corrects for this timing mismatch:

$$IGR = \frac{ROA \times b}{1 - ROA \times b}$$

While both of these definitions are correct if appropriate inputs are used, this latter formulation allows the analyst to rely upon published sources for ROA, and is therefore preferable for most classroom purposes.

Again, the *internal* growth rate represents the maximum rate at which the firm's sales and assets can grow using *only* internally generated funds (retained earnings). That is, it assumes that any increase in the level of assets can be funded strictly with a corresponding increase in shareholders' equity (retained earnings).

Obviously, the realized growth rate in sales is unknowable in advance, and is unlikely to match this theoretical optimum. If a profitable firm actually achieves positive sales growth (assuming less than 100% payout and keeping total liabilities fixed), then the balance sheet will necessarily be affected: total assets and shareholders' equity will increase by equal amounts, while liabilities remain unchanged. Consequently, if the growth rate in sales is positive but less than the IGR, the firm will experience a decrease in the debt ratio, potentially drifting away from its target capital structure over time.

Profitable sales growth *in excess* of the IGR would potentially be even more problematic; at this level of growth, retained earnings do not provide sufficient capital to fund the investment in additional assets needed. Without additional financing, the firm in this situation will quickly run into a cash crunch. In theory, at least, the IGR gives managers a tool for anticipating and preparing for such a situation.

Growing businesses are of course not precluded from raising additional capital to fund their growth. While the IGR represents the maximum rate at which the firm's assets can grow with *no* additional external capital, the *sustainable* growth rate concept relaxes that restriction. The question answered by the SGR is: how much growth can the firm support, assuming that it is able and willing to increase its use of debt financing, without increasing its debt ratio? That is, the implicit assumption of the SGR is that the firm is willing to take on new debt to fund its growth, but not beyond the point that it results in an increase in the debt ratio.

In terms of the balance sheet, this means that asset growth is allowed to be funded by equal percentage increases in both the shareholders' equity and total liability components, as opposed to shareholders' equity alone. The SGR, then is the rate at which *shareholders' equity* will increase year-over-year as a result of retained earnings:

$$SGR = \frac{EQUITY_1 - EQUITY_0}{EQUITY_0}$$

Given that the increase in equity comes from retained net income, we can make a substitution similar to the above:

$$SGR = \frac{NI_1 \times b}{EQUITY_0}$$

Following the same logic as the IGR derivation and using the usual definition of return on equity (ROE), we obtain the customary expression for the self-sustainable growth rate:

$$SGR = \frac{ROE \times b}{1 - ROE \times b}$$

## USING PERCENT-OF-SALES FORECAST TO ESTIMATE GROWTH RATES

Having been exposed to the percent-of-sales method for constructing pro forma financial statement models, students can be asked to determine a firm's internal and sustainable rates of growth directly, using such a model. In the process of doing this, they may well encounter inconsistencies with the simple mathematical models described above. The example below demonstrates the frustrations that students may encounter when trying to reconcile the result of the IGR/SGR formulas with the insights derived from a slightly more detailed model.

The following example is based upon the "Hoffman Inc." example provided in chapter 4 of Ross, Westerfield and Jordan (2021). It begins with the presentation of a very rudimentary set of financial statements, illustrated here in Figure 1.

In this simple example, the IGR and SGR are easily calculated as 9.64% and 21.35%, according to the formulas given above. If we apply the customary percent-of-sales approach to produce a set of pro forma statements for the upcoming year, it's easy to confirm the validity of these numbers. For the purposes of this example, we will assume that the tax rate remains at 21%, the payout ratio remains at 33.3%, and all costs and assets maintain their current size relative to sales. The level of debt is assumed to remain fixed. The resulting pro formas are illustrated in Figure 2.

**Figure 1**

**Hoffman Company (ORIGINAL)**

| Income Statement | | | Balance Sheet | |
|---|---|---|---|---|
| | **20X0** | | **Assets** | **20X0** |
| Sales | 500 | | Current | 200 |
| Costs | 416.5 | | Fixed | 300 |
| Taxable Income | 83.5 | | Total | 500 |
| Taxes @ 21% | 17.5 | | | |
| Net Income | 66.0 | | | |
| | | | **Liabs & Equity** | |
| Dividends | 22.0 | | Debt | 250 |
| Addition to RE | 44.0 | | Equity | 250 |
| | | | Total | 500 |
| | | | | |
| | | | **ROA** | **13.19%** |
| | | | **ROE** | **26.39%** |
| | | | | |
| | | **Internal Growth Rate** | | **9.64%** |
| | | **Sustainable Growth Rate** | | **21.35%** |

**Figure 2**

**Hoffman Company (9.64% Growth)**

**Income Statement**

| | 20X0 | 20X0% | 20X1 |
|---|---|---|---|
| Sales | 500 | 100% | 548.2 |
| Costs | 416.5 | 83.3% | 456.7 |
| Taxable Income | 83.5 | | 91.6 |
| Taxes | 17.5 | calc @ 21% | 19.2 |
| Net Income | 66.0 | | 72.3 |
| | | | |
| Dividends | 22.0 | | 24.1 |
| Addition to RE | 44.0 | | 48.2 |

**Balance Sheet**

| Assets | 20X0 | 20X0% | 20X1 |
|---|---|---|---|
| Current | 200 | 40% | 219.288 |
| Fixed | 300 | 60% | 328.932 |
| Total | 500 | | 548.22 |
| | | | |
| **Liabs & Equity** | | | |
| Debt | 250 | fixed | 250 |
| Equity | 250 | calc | 298.2 |
| Total | 500 | | 548.2 |
| | | | |
| **External Financing Needed** | | | **0.0** |

Here, sales are assumed to grow at a rate of 9.64% year-over-year (the calculated IGR), and, as expected, the firm will experience neither a capital surplus nor deficit in this scenario; the amount of external financing needed is \$0. Once students have built a working spreadsheet model similar to Figure 2, they can experiment by changing the growth rate assumption to see the effect of growth on the firm's financial requirements. They should find that growth rates higher than the IGR result in a positive EFN (a capital deficit), and growth rates lower than the IGR yield a negative EFN (a capital surplus). Another worthwhile exercise is to invite students to use the Goal Seek feature of Microsoft Excel to verify the results of the IGR formula.

If we modify the spreadsheet a bit such that the debt-to-equity ratio remains fixed (debt is allowed to grow at the same rate as sales), students can see that the additional capital supports higher levels of growth. Figure 3 illustrates such a model using an assumed growth rate of 21.35% (the calculated SGR). When debt is allowed to increase along with sales, we again find that the firm can support growth at this rate without either a deficit or surplus of capital (EFN=0). As before, students can experiment with higher and lower growth rates, or verify the accuracy of the SGR formula using Goal Seek.

**Figure 3**

**Hoffman Company (21.35% Growth, constant D/E)**

| Income Statement | | | | | Balance Sheet | | | |
|---|---|---|---|---|---|---|---|---|
| | 20X0 | 20X0% | 20X1 | | Assets | 20X0 | 20X0% | 20X1 |
| Sales | 500 | 100% | 606.7 | | Current | 200 | 40% | 242.69 |
| Costs | 416.5 | 83.3% | 505.4 | | Fixed | 300 | 60% | 364.04 |
| Taxable Income | 83.5 | | 101.3 | | Total | 500 | | 606.73 |
| Taxes @ 21% | 17.5 | calc @ 21% | 21.3 | | | | | |
| Net Income | 66.0 | | 80.0 | | | | | |
| | | | | | Liabs & Equity | | | |
| Dividends | 22.0 | | 26.7 | | Debt | 250 | 100%D/E | 303.36 |
| Addition to RE | 44.0 | | 53.4 | | Equity | 250 | calc | 303.36 |
| | | | | | Total | 500 | | 606.73 |
| | | | | | External Financing Needed | | | 0.00 |

## PROBLEMS WITH THE POPULAR MODEL

The preceding examples reconcile correctly with the popular models of IGR and SGR respectively, only because they are built upon the assumption that all costs, all asset accounts, and all current liabilities will increase at the same rate as top line sales. Only if this rather strong assumption is upheld can we say that the maximum rate of sales growth that Hoffman can support without external financing is 9.64%. Therein lies the problem with the popular models; such an assumption is unlikely to comport with reality, or with typical managerial practice. For nearly every business, some costs are variable and some are fixed, so total operating costs will not scale in direct proportion to revenue. Similarly, working capital and fixed asset requirements are unlikely to grow in that manner as well, given the "lumpiness" of capital expenditures, and the potential availability of excess capacity. Even the most rudimentary effort to develop a set of pro forma financial statements should account for these considerations.

Furthermore, even modest deviations from the equal-growth assumption result in economically significant changes to the "true" IGR and SGR, throwing the validity of the popular model into doubt. Suppose, for example, that we revise the Hoffman forecast again, accounting for the assumption that 40% of the firm's total costs are fixed; that is, the ratio of fixed costs to total costs is 0.40. The resulting forecast is presented in Figure 4. Note that this change would not affect the value of either IGR or SGR calculated according to the popular model; they would still be 9.64% and 21.35% respectively. However, in Figure 4, we find (by adjusting the assumed growth rate) that with no increase in debt, the firm can now sustain growth of approximately 17.88%. That is, Hoffman's true IGR is 17.88%. Similarly, the firm's true SGR of 118.09% is obtained in Figure 5 by assuming that debt is allowed to grow in order to maintain a constant debt-to-equity ratio, and then finding the growth rate at which no *additional* external financing is needed (EFN = 0).

**Figure 4**

**Hoffman Company (40% FC/TC, 17.88% Growth)**

| **Income Statement** | **20X0** | **20X0%** | **20X1** |
|---|---|---|---|
| Sales | 500 | 100% | 589.38 |
| Fixed Costs | 160 | fixed | 160.00 |
| Variable Costs | 240 | 48% | 282.90 |
| Total Costs | 400 | | 442.90 |
| Taxable Income | 100 | | 146.48 |
| Taxes | 21 | calc @ 21% | 30.76 |
| Net Income | 79 | | 115.72 |
| | | | |
| Dividends | 26.33 | | 26.33 |
| Addition to RE | 52.67 | | 89.38 |

| **Balance Sheet** | **20X0** | **20X0%** | **20X1** |
|---|---|---|---|
| **Assets** | | | |
| Current | 200 | 40% | 235.75 |
| Fixed | 300 | 60% | 353.63 |
| Total | 500 | | 589.38 |
| | | | |
| **Liabs & Equity** | | | |
| Debt | 250 | fixed | 250.00 |
| Equity | 250 | calc | 339.38 |
| Total | 500 | | 589.38 |
| | | | |
| **External Financing Needed** | | | **0.0** |

**Figure 5**

**Hoffman Company (40% FC/TC, 100% D/E, 118.09% Growth)**

| **Income Statement** | **20X0** | **20X0%** | **20X1** |
|---|---|---|---|
| Sales | 500 | 100% | 1090.45 |
| Fixed Costs | 160 | fixed | 160.00 |
| Variable Costs | 240 | 48% | 523.42 |
| Total Costs | 400 | | 683.42 |
| Taxable Income | 100 | | 407.03 |
| Taxes | 21 | calc @ 21% | 85.48 |
| Net Income | 79 | | 321.56 |
| | | | |
| Dividends | 26.33 | | 26.33 |
| Addition to RE | 52.67 | | 295.22 |

| **Balance Sheet** | **20X0** | **20X0%** | **20X1** |
|---|---|---|---|
| **Assets** | | | |
| Current | 200 | 40% | 436.18 |
| Fixed | 300 | 60% | 654.27 |
| Total | 500 | | 1090.45 |
| | | | |
| **Liabs & Equity** | | | |
| Debt | 250 | 100% D/E | 545.22 |
| Equity | 250 | calc | 545.22 |
| Total | 500 | | 1090.45 |
| | | | |
| **External Financing Needed** | | | **0.0** |

To summarize: the popular models produced estimates of IGR and SGR of 9.64% and 21.35% respectively. We have shown, though, that under the simple, realistic assumption that 40% of costs are fixed, the *actual* growth limits are 17.88% and 118.09%. While these results (particularly SGR) are a bit extreme, it is clear that the presence of fixed costs in the firm's cost structure has a non-trivial effect on its ability to sustain growth. This observation serves as an illustration of the concept of operating leverage, which we will address below.

**Alternative Models of IGR and SGR**

It is possible to derive relatively compact formulas for IGR and SGR which incorporate fixed costs and variable costs, presented below (derivation in appendix):

$$\boldsymbol{IGR} = \left[\frac{\boldsymbol{A - FC(1-T) - DIV}}{\boldsymbol{A - (S-VC)(1-T)}}\right] - \mathbf{1} \quad (1)$$

$$\boldsymbol{SGR} = \left[\frac{\boldsymbol{EQ - FC(1-T) - DIV}}{\boldsymbol{EQ - (S-VC)(1-T)}}\right] - \mathbf{1} \quad (2)$$

Here, the reported amounts of fixed costs and variable costs are denoted as *FC* and *VC* respectively. Note that interest expense, depreciation, and any other noncash charges would be included as part of *FC*. The most recent observations of total assets, shareholders' equity, and sales are represented by *A, EQ,* and *S* respectively. *DIV* denotes the amount of dividends to be paid, and *T* the tax rate. While these last two inputs are forward-looking, near-term estimates are generally reliable when it comes to dividend policy and the corporate income tax rate. Substituting the relevant values from the Hoffman example into the preceding equations, we obtain the expected results:

$$IGR = \left[\frac{500-160(1-0.21)-26.33}{500-(500-240)(1-0.21)}\right] - 1 = 0.1788 = 17.88\%$$

$$SGR = \left[\frac{250-160(1-0.21)-26.33}{250-(500-240)(1-0.21)}\right] - 1 = 1.1809 = 118.09\%$$

The above formulations for a firm's internal and sustainable growth rates are obviously a bit more involved than the popular models; however, they provide a relatively quick, back-of-the-envelope method for managers to assess their firm's ability to accommodate sales growth, without resorting to a full-fledged spreadsheet model.

By recognizing that $S - FC - VC =$ earnings before interest and taxes (EBIT), it is possible to reformulate the numerator of the SGR expression as follows:

$$SGR = \frac{EBIT_0(1-T) - DIV_1}{EQ_0 - (S-VC)(1-T)} \quad (3)$$

It should be noted that firms do not generally release detailed information about their cost structures to the public, so precise values for *FC* and *VC* may not be available to external analysts. The IGR and SGR concepts themselves are primarily useful in the context of internal planning, in which this limitation would not be an issue. If external analysts seek to estimate IGR or SGR, industry averages could potentially be used, and would be preferable to the assumption of 100% variable costs which is implicit in the popular model. In the context of a forecasting project or discussion of the concept of sustainable growth, the important takeaway for students is to recognize the limitations of the popular models of IGR and SGR.

## OPERATING LEVERAGE AND THE SUSTAINABLE GROWTH RATE

As we have demonstrated, the popular models are correct only when fixed costs and total costs are equal; therefore, if the ratio of fixed costs to total costs is not generally equal to (or very close to) one, the popular models will not be accurate. To assess the real-world significance of this problem, it would be natural to examine the ratio of fixed costs to total costs for a sample of actual firms. However, public firms in the United States do not generally disclose details of their cost structures, so a direct measurement of this ratio is impractical. We can, though, evaluate the relative extent of firms' fixed costs indirectly, using the concept of operating leverage.

According to Block, et al (2022), "operating leverage reflects the extent to which fixed assets and associated fixed costs are utilized in the business." Operating leverage is measured by the Degree of Operating Leverage (*DOL*), defined by Gup (1986) as "the percentage change in EBIT that will result from a 1 percent change in sales revenue". The concept is frequently operationalized as:

$$DOL = \frac{S - VC}{S - VC - FC}$$

In this formula, *DOL* is defined in terms of sales, variable costs, and fixed costs. The only difference between the numerator and denominator of this expression is the amount of fixed costs, *FC*. The assumption that all costs are variable (implicit in the popular model of SGR) is equivalent to assuming that $FC = 0$, in which case *DOL* would be equal to 1. Consequently, for any firm for which *DOL* is appreciably different from 1, the traditional formulation of SGR will be inappropriate. For example, for the fictitious Hoffman Company described above, the initial value of *DOL* would be:

$$DOL = \frac{500 - 240}{500 - 240 - 160} = 2.60$$

As demonstrated above, when Hoffman is assumed to have fixed costs amounting to 40% of total costs (equating to *DOL* of 2.60), the popular models of IGR and SGR fail to produce useful results. But, is this degree of operating leverage typical of real-world firms? Again, without access to firm-level data for the values of FC and VC, we cannot use this formulation for cross-sectional analysis. Techniques for estimation of *DOL* have been the subject of extensive research (Dugan and Shriver 1989, for example). However, for the present purpose it is possible to use a more fundamental definition of *DOL* (Brealey, et al 2020):

$$DOL = \frac{\text{percentage change in profits}}{\text{percentage change in sales}}$$

Because this formulation relies only on publicly available income statement data, it is possible to produce *ex-post* estimates of *DOL* for publicly traded firms. In the following section, we assess the real-world significance of the issues highlighted in this study, using estimates of *DOL* calculated in this way.

## THE DEGREE OF OPERATING LEVERAGE IN PRACTICE

The Bloomberg Information System provides pre-computed estimates of the *DOL* for public firms according to the preceding definition, using EBIT as the measure of profit. It should be noted that Bloomberg's procedure modifies the simple calculation above in two ways: first, when the numerator and denominator of this fraction are of opposing signs (for example, EBIT decreases in spite of an increase in sales), the observation is discarded. When Sales and EBIT move in opposing directions, it is taken as an indication of a period of structural change which would render the interpretation of *DOL* meaningless. Second, when both numerator and denominator are negative, the observation of *DOL* is encoded as negative. In these cases, the sign serves only as an indicator of a decline in both sales and profits, not as a signal of an inverse relationship between the two (Bloomberg Finance L.P, n.d.). Consequently, we use the absolute value of Bloomberg's reported *DOL* in the following analysis.

We chose to use the firms of the Standard and Poor's Midcap 400 as the basis for this analysis. This is a set of 400 publicly traded U.S. corporations with market capitalizations ranging from approximately $4.6 billion to $12.7 billion as of May 2023. This index was chosen because the firms in this size range "have successfully navigated the challenges specific to small companies", while at the same time are "not so large that continued growth is unattainable" (S&P Dow Jones Indices 2023). In short, these are firms for which growth is a concern, but not the only concern.

We obtain from Bloomberg the pre-computed estimates of DOL for the firms of the Midcap 400 index as of the end of calendar years 2018 through 2022. This resulted in 1085 data points. As mentioned above, only the unsigned absolute values are used in the analysis.

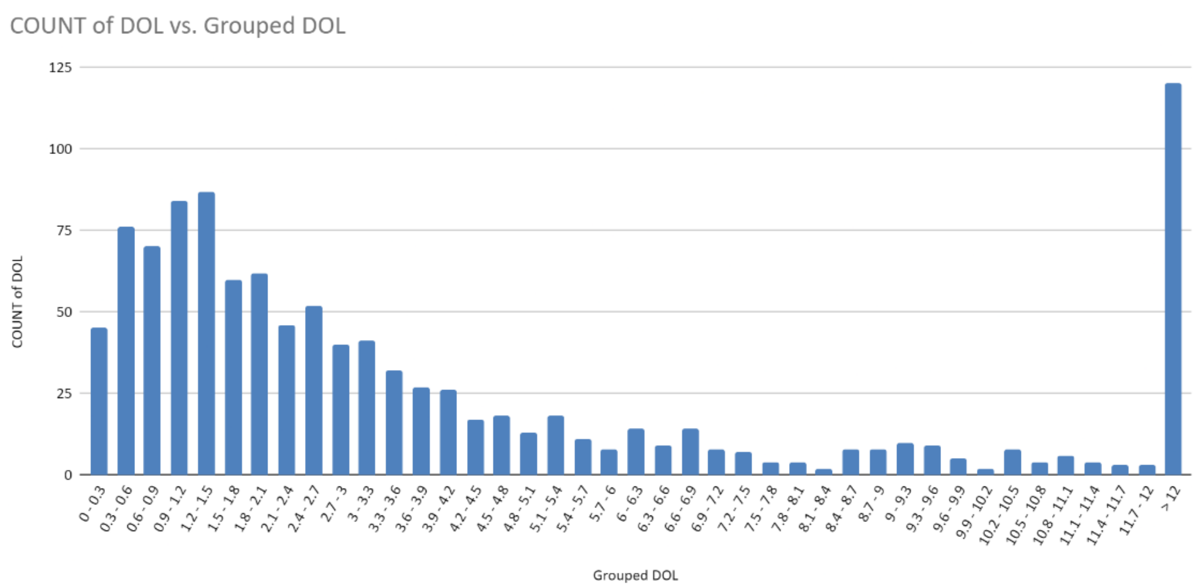
The issue at hand is whether or not the implicit assumption of DOL = 1 is generally valid, in which case the traditional formulations of IGR and SGR would produce reliable results. Within the full sample, the mean DOL was 11.07, with a standard deviation of 46.56. The sample median was 2.45; notably, not far from the value of 2.60 exhibited by the fictitious "Hoffman" example above. Reported values for DOL ranged from 0.0076 to approximately 763. Summary statistics are provided in Table 1, and a histogram illustrating the distribution of the sample is provided as Figure 6. The 95% confidence interval for the mean DOL spans the range from approximately 8.3 to 13.8. That is, we can say with 95% certainty that the mean DOL of midsize U.S. firms falls within this range; nowhere near the critical value of 1.0.

To minimize the effect of extreme (outlier) values in the sample, we repeated the calculations after eliminating the 25 largest and 25 smallest observations. This produced a subsample of 1035 values, having a mean of 5.46 and a standard deviation of 9.2. Again, summary statistics are presented in the rightmost column of Table 1. The resulting 95% confidence interval based on this sample was [4.9, 6.0]; again, the critical value 1.0 does not fall within this range. Based on this, we can confidently assert that it is not typical for firms in this category to exhibit a degree of operating leverage that is close to 1.0, and therefore the popular models for IGR and SGR should be used cautiously in practice, if at all. Revisiting the analysis using the firms of the S&P 500 index did not materially alter these findings. Also, while it may be possible to use various approaches to develop estimates for DOL from financial statement data (Dugan and Shriver 1989), it seems unlikely (based on the results of the cited research) that such estimates would be typically close enough to 1.0 to render the traditional models of SGR and IGR viable.

**Table 1**

| | **MidCap 400 - DOL** | **MidCap 400 - DOL (excluding outliers)** |
|---|---|---|
| Mean | 11.07 | 5.46 |
| Median | 2.45 | 2.45 |
| Standard Deviation | 46.56 | 9.20 |
| Minimum | 0.0076 | 0.1653 |
| Maximum | 762.88 | 85.36 |
| Count | 1085 | 1035 |

**Figure 6**



## USING THE DEGREE OF OPERATING LEVERAGE TO ESTIMATE THE SUSTAINABLE GROWTH RATE

If an external analyst lacks actual values for *VC* and *FC*, but has a reasonable estimate of *DOL* (from either direct observation as above or from another estimation approach), our improved models for IGR and SGR can be rewritten to eliminate the unavailable data items. Let *TC* represent total costs, i.e. *TC* is the sum of *FC* and *VC*. Then,

$$VC = S - DOL(S - TC)$$

and

$$FC = TC - VC = (TC - S)(1 - DOL)$$

Substituting these expressions into the previous formulas for IGR and SGR, we get:

$$\boldsymbol{IGR} = \left[\frac{\boldsymbol{A} - (\boldsymbol{TC} - \boldsymbol{S})(1 - \boldsymbol{DOL})(1 - \boldsymbol{T}) - \boldsymbol{DIV}}{\boldsymbol{A} - (\boldsymbol{DOL}(\boldsymbol{TC} - \boldsymbol{S}))(1 - \boldsymbol{T})}\right] - \mathbf{1} \qquad (4)$$

and:

$$\boldsymbol{SGR} = \left[\frac{\boldsymbol{EQ} - (\boldsymbol{TC} - \boldsymbol{S})(1 - \boldsymbol{DOL})(1 - \boldsymbol{T}) - \boldsymbol{DIV}}{\boldsymbol{EQ} - (\boldsymbol{DOL}(\boldsymbol{TC} - \boldsymbol{S}))(1 - \boldsymbol{T})}\right] - \mathbf{1} \qquad (5)$$

While these expressions are somewhat less elegant than equations (1) and (2), they have the advantage of not relying upon potentially unavailable values for *FC* and *VC* (the value for total operating cost *TC* is likely to be more readily available). Instead, the growth limits are calculated on the basis of *DOL*, which could be estimated by direct observation of the relationship between the changes in profits and sales over time (as in the analysis above), or one of the other methods documented in the literature.

## CONCLUSION

The self-sustainable growth rate concept is an important one for students and practitioners in the field of financial management. In the classroom, it helps to illustrate the fact that for nearly all firms, rapid growth in revenue is a double-edged sword; while sales growth normally translates to higher earnings, it also brings the need for additional (costly) assets. The traditional formulations of IGR and SGR can be useful to help students understand the fundamental connections between revenue growth and the firm's capital investment, debt, and dividend policies.

However, as has been shown above, the traditional models depend critically upon the assumption that all costs and all asset requirements grow at the same rate as revenue, which is unlikely to be the case for most firms. And we show that realistic deviations from this assumption can cause the calculated IGR and SGR to vary widely from their "true" values, to the point that the traditional models are of questionable utility for any practical purpose.

In this paper we present revised formulations for IGR and SGR which take into account the firm's structure of fixed and variable costs (and equivalently, its degree of operating leverage). While these formulations are a bit less elegant than the popular models, they have the advantage of being based upon far more realistic assumptions, and thereby producing more reliable results. These models are useful in at least two ways. First, in corporate finance courses, they can help to resolve students' confusion regarding the limitations of the popular model, and to understand the factors that limit firms' ability to accommodate growth. Second, analysts and managers may find the revised models useful for anticipating and planning for capital requirements in future periods.

In the practice of financial planning, no (practical) formula can take the place of a detailed, integrated financial statement model for the purpose of developing expectations and plans for future periods. It is hoped, though, that this note will serve as a caution for instructors and students of corporate finance to be aware of the limitations of the popular IGR/SGR models, and to consider the many other factors that have an influence on a firm's self-sustainable rate of sales growth.

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## APPENDIX

We can define external financing needed ($EFN_1$) as follows:

$$EFN_1 = Assets_1 - Debt_1 - EQ_1$$

We make the assumption that all assets grow at the same rate as sales, $g$:

$$Assets_1 = Assets_0(1+g)$$

Shareholders' equity at the end of period, $EQ_1$, can be described in terms of retained earnings $RE$:

$EQ_1 = EQ_0 + \Delta RE$, where $\Delta RE = [(S - VC)(1+g) - FC](1-T) - DIV_1$.

Therefore,

$$EQ_1 = EQ_0 + [(S - VC)(1+g) - FC](1-T) - DIV_1$$

If we substitute this expression into the previous formula for $EFN_1$, we obtain

$$EFN_1 = Assets_0(1+g) - Debt_1 - EQ_0 - [(S - VC)(1+g) - FC](1-T) + DIV_1$$

Now, we want to find the growth rate, $g$, for which $EFN_1$= 0 (according to the definition of *IGR*). In order to do this, we have to solve for $g$ the following linear equation:

$$0 = Assets_0(1+g) - Debt_1 - EQ_0 - (S - VC)(1+g)(1-T) + FC(1-T) + DIV_1$$

Gathering the terms with $g$ on the left side, we will have the following:

$$(S - VC)(1-T)(1+g) - Assets_0(1+g) = -Debt_1 - EQ_0 + FC(1-T) + DIV_1$$

Multiply both sides by (−1):

$$Assets_0(1+g) - (S - VC)(1-T)(1+g) = Debt_1 + EQ_0 - FC(1-T) - DIV_1$$

Factor out $(1+g)$ on the left:

$$[Assets_0 - (S - VC)(1-T)](1+g) = Debt_1 + EQ_0 - FC(1-T) - DIV_1$$

Now, solve for $(1+g)$:

$$(1+g) = \frac{Debt_1 + EQ_0 - FC(1-T) - DIV_1}{Assets_0 - (S - VC)(1-T)}$$

Now, we can use the fundamental balance sheet identity $Debt_0 + EQ_0 = Assets_0$

Since for calculating *IGR* we are using the assumption that the total liabilities are fixed, that is, the company does not acquire any new debt, then $Debt_0 = Debt_1$, therefore we can rewrite the previous equation as $Debt_1 + EQ_0 = Assets_0$. Using this last expression, we can simplify the above formula for $(1+g)$ as follows:

$$1 + g = \frac{Assets_0 - FC(1-T) - DIV_1}{Assets_0 - (S - VC)(1-T)},$$

which results in the following formula for $IGR$:

$$IGR = g = \frac{Assets_0 - FC(1-T) - DIV_1}{Assets_0 - (S - VC)(1-T)} - 1$$

To derive the formula for SGR, we go back to the equation

$$0 = Assets_0(1+g) - Debt_1 - EQ_0 - (S - VC)(1+g)(1-T) + FC(1-T) + DIV_1,$$

and make the assumption that debt, just like assets, grows at the same rate as sales, $g$:

$$Debt_1 = Debt_0(1+g)$$

Substituting this expression into the equation, we get the following:

$$0 = Assets_0(1+g) - Debt_0(1+g) - EQ_0 - (S - VC)(1+g)(1-T) + FC(1-T) + DIV_1$$

which can be rewritten as

$$0 = (Assets_0 - Debt_0)(1+g) - EQ_0 - (S - VC)(1+g)(1-T) + FC(1-T) + DIV_1$$
$$= EQ_0(1+g) - EQ_0 - (S - VC)(1+g)(1-T) + FC(1-T) + DIV_1$$

If we solve this equation for $g$, the resulting formula is the expression for calculating $SGR$:

$$SGR = g = \frac{EQ_0 - FC(1-T) - DIV_1}{EQ_0 - (S - VC)(1-T)} - 1$$

Note: we can further simplify the above equation as

$$0 = EQ_0(g) - (S - VC)(1+g)(1-T) + FC(1-T) + DIV_1$$

Gathering terms with $g$ on the right and factoring $g$ out, we get:

$$(S - VC)(1-T) - FC(1-T) - DIV_1 = [EQ_0 - (S - VC)(1-T)](g),$$

or, equivalently,

$$(S - VC - FC)(1-T) - DIV_1 = [EQ_0 - (S - VC)(1-T)](g)$$

The expression $(S - VC - FC)$ defines earnings before interest and taxes (EBIT), so an alternative formula for $SGR$ is:

$$SGR = g = \frac{EBIT_0(1-T) - DIV_1}{EQ_0 - (S - VC)(1-T)}$$